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LETTER TO THE EDITOR

Long-range radiative interaction of magnetic moments in an electromagnetic wave

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Abstract. The induced oscillations of magnetic moments result in the appearance of time-averaged, long-range radiative forces. These forces are square-law ones to the field amplitude and decrease inversely proportionally to the distance between the moments. The magnetic moments are considered to be classical or quantum and the electromagnetic waves are considered to be classical.

Recently it was found that the induced oscillations of small particles (gas bubbles and solid corpuscles) in compressible liquid under the action of sound waves result in the appearance of long-range radiative forces [1-4]. These time-averaged forces are square-law ones to the field amplitude and decrease inversely proportionally to the distance between the particles. The relative motion of two particles was investigated in [4]. Doinikov and Zavtrak [4] have explained the phenomenon of the bound state formation of the gas bubbles which were observed in [5]. Finally, it was found in [6] that the induced dipole oscillations of the charged particles in an electromagnetic field lead to the appearance of analogous radiative forces. These forces are caused by the secondary radiation of the charges.

In this letter the classical and quantum theory of the radiative interaction of two magnetic moments in electromagnetic waves is suggested. The external fields result in force moments and consequently lead to the precession of magnetic moments. The oscillating magnetic moments radiate the secondary electromagnetic waves which give rise to the forces mentioned above. The particles (magnetic moments) are considered to be classical or quantum and the electromagnetic waves are considered to be classical.

Such possible interdisciplinary transfer is based on the formal analogy of corresponding equations as is usual in physics [7-9]. It is also clear that the term particle itself has not the same significance in fluid mechanics as in electrodynamics.

Let us consider two magnetic moments in an external magnetic wave $\boldsymbol{H}_{\text{ext}} = \boldsymbol{H}_0 \cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{r})$, where $\boldsymbol{H}_0$ is the amplitude of the magnetic intensity vector, ω is the cyclic frequency and $\boldsymbol{k}$ is the wavevector. Let $\boldsymbol{r}_{1,2}$ be the radius vectors of their positions. The magnetic moments $\boldsymbol{\mu}_{1,2}$ are expressed through the usual impulsive moments $\boldsymbol{L}_{1,2}$ (as $\hbar = 1$)

$$\boldsymbol{\mu}_{1,2} = g_{1,2}\mu_{\text{B}1,2}\boldsymbol{L}_{1,2}$$

where $\mu_{\text{B}1,2}$ are Bohr magnetons [10] and $g_{1,2}$ are g-factors. The magnetic field $\boldsymbol{H}(\boldsymbol{r}, t)$ results in the force moments and consequently leads to the precession [11]

$$\dot{\boldsymbol{L}}_{1,2} = [\boldsymbol{\mu}_{1,2}, \boldsymbol{H}(\boldsymbol{r}_{1,2}, t)].$$

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For the small oscillations one obtains

$$\delta\dot{\boldsymbol{\mu}}_{1,2} = g_{1,2}\mu_{B1,2}[\boldsymbol{\mu}_{1,2}, \boldsymbol{H}(\boldsymbol{r}_{1,2}, t)]. \tag{1}$$

In this equation $\boldsymbol{\mu}_{1,2}$ can be considered to be constant vectors.

The oscillating magnetic moments radiate the electromagnetic waves. At the far zone $k|\boldsymbol{r}_1 - \boldsymbol{r}_2| \gg 1$ (we examine only the far zone because the most interesting results occur at this limit) the scattering magnetic intensity vectors are [11]

$$\boldsymbol{H}_{r_{1,2}} = \frac{1}{c^2 R_{1,2}} [[\delta\ddot{\boldsymbol{\mu}}_{1,2}, \boldsymbol{n}_{1,2}], \boldsymbol{n}_{1,2}]. \tag{2}$$

The unit vectors $\boldsymbol{n}_{1,2}$ give the direction of radiation, $R_{1,2}$ are the propagation distances, c is the velocity of light. The expression (2) should be calculated at delayed moments of time.

The resulting magnetic field acting at the particle is the sum of the external wave and the wave created by the neighbouring oscillating moment. Therefore

$$\begin{aligned} \delta\dot{\boldsymbol{\mu}}_1(t) &= g_1\mu_{B1}[\boldsymbol{\mu}_1, \boldsymbol{H}_0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_1) + \boldsymbol{H}_{r_2}(\boldsymbol{r}_1, t)] \\ \delta\dot{\boldsymbol{\mu}}_2(t) &= g_2\mu_{B2}[\boldsymbol{\mu}_2, \boldsymbol{H}_0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_2) + \boldsymbol{H}_{r_1}(\boldsymbol{r}_2, t)]. \end{aligned} \tag{3}$$

The solution of this system can be represented as

$$\delta\boldsymbol{\mu}_1(t) = \delta\boldsymbol{\mu}_{10}(t) + \delta\boldsymbol{\mu}_{11}(t) \qquad \delta\boldsymbol{\mu}_2(t) = \delta\boldsymbol{\mu}_{20}(t) + \delta\boldsymbol{\mu}_{21}(t) \tag{4}$$

where

$$\begin{aligned} \delta\boldsymbol{\mu}_{10}(t) &= \frac{g_1\mu_{B1}}{\omega}[\boldsymbol{\mu}_1, \boldsymbol{H}_0]\sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_1) \\ \delta\boldsymbol{\mu}_{20}(t) &= \frac{g_2\mu_{B2}}{\omega}[\boldsymbol{\mu}_2, \boldsymbol{H}_0]\sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}_2) \\ \delta\boldsymbol{\mu}_{11}(t) &= \frac{g_1\mu_{B1}}{c^2 l}[\boldsymbol{\mu}_1, [[\delta\dot{\boldsymbol{\mu}}_{20}(t - l/c), \boldsymbol{n}], \boldsymbol{n}]] + \mathrm{O}\left(\frac{1}{l^2}\right) \\ \delta\boldsymbol{\mu}_{21}(t) &= \frac{g_2\mu_{B2}}{c^2 l}[\boldsymbol{\mu}_2, [[\delta\dot{\boldsymbol{\mu}}_{10}(t - l/c), \boldsymbol{n}], \boldsymbol{n}]] + \mathrm{O}\left(\frac{1}{l^2}\right) \end{aligned} \tag{5}$$

and $\boldsymbol{n} = \boldsymbol{l}/l$, $\boldsymbol{l} = \boldsymbol{r}_2 - \boldsymbol{r}_1$.

The energy of the first moment in a magnetic field is $W_1 = -((\boldsymbol{\mu}_1 + \delta\boldsymbol{\mu}_1)\cdot\boldsymbol{H}_1(\boldsymbol{r}_1, t))$ where $\boldsymbol{H}_1(\boldsymbol{r}_1, t) = \boldsymbol{H}_{ext}(\boldsymbol{r}_1, t) + \boldsymbol{H}_{r_2}(\boldsymbol{r}_1, t)$. Analogously $W_2 = -((\boldsymbol{\mu}_2 + \delta\boldsymbol{\mu}_2)\cdot\boldsymbol{H}_2(\boldsymbol{r}_2, t))$ where $\boldsymbol{H}_2(\boldsymbol{r}_2, t) = \boldsymbol{H}_{ext}(\boldsymbol{r}_2, t) + \boldsymbol{H}_{r_1}(\boldsymbol{r}_2, t)$.

The time-averaged forces acting on the magnetic moments can be usually calculated as [12]

$$F_{1i} = \left\langle (\mu_{1j} + \delta\mu_{1j})\frac{\partial H_{1j}(\boldsymbol{r}_1, t)}{\partial x_{1i}} \right\rangle \qquad F_{2i} = \left\langle (\mu_{2j} + \delta\mu_{2j})\frac{\partial H_{2j}(\boldsymbol{r}_2, t)}{\partial x_{2i}} \right\rangle. \tag{6}$$

The calculations give

$$\boldsymbol{F}_1 = S(\boldsymbol{k} + k\boldsymbol{n})\sin(kl + \boldsymbol{k}\cdot\boldsymbol{l}) \qquad \boldsymbol{F}_2 = S(\boldsymbol{k} - k\boldsymbol{n})\sin(kl - \boldsymbol{k}\cdot\boldsymbol{l}) \tag{7}$$

where

$$\begin{aligned} S = \frac{g_1 g_2 \mu_{B1}\mu_{B2}}{2c^2 l}[&H_0^2(\boldsymbol{\mu}_1\cdot\boldsymbol{n})(\boldsymbol{\mu}_2\cdot\boldsymbol{n}) - (\boldsymbol{\mu}_2\cdot\boldsymbol{H}_0)(\boldsymbol{\mu}_1\cdot\boldsymbol{n})(\boldsymbol{H}_0\cdot\boldsymbol{n}) \\ &- (\boldsymbol{\mu}_1\cdot\boldsymbol{H}_0)(\boldsymbol{\mu}_2\cdot\boldsymbol{n})(\boldsymbol{H}_0\cdot\boldsymbol{n}) + (\boldsymbol{n}\cdot\boldsymbol{H}_0)^2(\boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2)]. \end{aligned}$$

It can be seen that the presence of external electromagnetic waves results in the appearance of long-range ($\sim l^{-1}$) radiative forces. This effect is analogous to the corresponding effects arising in classical acoustics.

It follows from expression (7) that the radiative forces consist of two components. One type has $\boldsymbol{l}$-direction and the second type has $\boldsymbol{k}$-direction. The sum $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ is not equal to zero. To our knowledge this result was found first in [1].

We should like to note that the forces (7) correspond to the action of the external waves (which are the sums of the incident wave and the wave created by the neighbouring moment) on the magnetic moments. Now to the self-interaction of the radiation field of the particle on itself. This question was considered in [11]. It was found [11, pp 245-6] that the resulting self-interaction force of the magnetic moment is equal to zero.

Let us try to evaluate the forces (7). At far zone $kl \gg 1$ the relation of the radiative force (7) to the usual static force of interaction is proportional to the parameter $\varepsilon = (kl)^3(\Omega_L/\omega)^2$. Here $\Omega_L = \mu_B H_0$ is the Larmor frequency in static magnetic field of magnitude H_0. At near zone $kl \ll 1$ the calculations give $\varepsilon = (\Omega_L/\omega)^2$.

Let us consider two quantum magnetic moments which are placed in an electromagnetic wave. The Hamiltonian of such a system is given by [10]

$$\hat{H}(t) = -g_1\mu_{B1}(\tfrac{1}{2}\hat{\boldsymbol{\sigma}}_1 \cdot \boldsymbol{H}(\boldsymbol{r}_1, t)) - g_2\mu_{B2}(\tfrac{1}{2}\hat{\boldsymbol{\sigma}}_2 \cdot \boldsymbol{H}(\boldsymbol{r}_2, t)) \tag{8}$$

where $\hat{\boldsymbol{\sigma}}_{1,2}$ are the Pauli spin matrices (we suppose that the particles have half-integral spins). The operators of magnetic moments are $\hat{\boldsymbol{\mu}}_{1,2} = g_{1,2}\mu_{B1,2}\hat{\boldsymbol{\sigma}}_{1,2}/2$.

The solution of the Schrödinger equation ($\hbar = 1$)

$$\mathrm{i}|\dot{\psi}(t)\rangle = \hat{H}(t)|\psi(t)\rangle \tag{9}$$

can be represented as

$$|\psi(t)\rangle = \hat{U}_1(t)\hat{U}_2(t)|\psi_0\rangle \tag{10}$$

where the unitary operators $\hat{U}_{1,2}(t)$ are

$$\hat{U}_{1,2}(t) = \frac{1 + \mathrm{i}\boldsymbol{a}_{1,2}(t) \cdot \hat{\boldsymbol{\sigma}}_{1,2}}{\sqrt{1 + a_{1,2}^2(t)}}. \tag{11}$$

In expression (11) $\boldsymbol{a}_{1,2}$ are Fedorov's vector parameters of a rotation group [13]. With an accuracy to the first order in field amplitude one obtains

$$\dot{\boldsymbol{a}}_{1,2}(t) = \tfrac{1}{2}g_{1,2}\mu_{B1,2}\boldsymbol{H}(\boldsymbol{r}_{1,2}, t). \tag{12}$$

The radiative forces acting at the magnetic moments can be calculated as [10]

$$\boldsymbol{F}_{1,2} = g_{1,2}\mu_{B1,2}\left\langle \psi(t) \left| \frac{\hat{\boldsymbol{\sigma}}_{1,2}}{2} \cdot \frac{\partial \boldsymbol{H}(\boldsymbol{r}_{1,2}, t)}{\partial \boldsymbol{r}_{1,2}} \right| \psi(t) \right\rangle. \tag{13}$$

Let us take into account that the resulting magnetic field acting on the particle is the sum of the external wave and the wave created by neighbouring moment. Then

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{r}_1, t) &= \boldsymbol{H}_0 \cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{r}_1) + \boldsymbol{H}_{r_2}(\boldsymbol{r}_1, t) \\ \boldsymbol{H}(\boldsymbol{r}_2, t) &= \boldsymbol{H}_0 \cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{r}_2) + \boldsymbol{H}_{r_1}(\boldsymbol{r}_2, t). \end{aligned} \tag{14}$$

At the far zone $kl \gg 1$ the scattering waves are (the electromagnetic waves are considered to be classical):

$$\boldsymbol{H}_{r_{1,2}} = \frac{1}{c^2 l}[[\ddot{\boldsymbol{m}}_{1,2}(t - l/c), \boldsymbol{n}], \boldsymbol{n}] \tag{15}$$

with

$$m_{1,2}(t)=\langle\psi(t)|\mu_{1,2}|\psi(t)\rangle=g_{1,2}\mu_{B1,2}\langle\psi(t)|\tfrac{1}{2}\hat{\sigma}_{1,2}|\psi(t)\rangle. \quad (16)$$

It follows from (9) and (16) that

$$\dot{m}_{1,2}(t)=g_{1,2}^2\mu_{B1,2}^2\langle\psi(t)|[\tfrac{1}{2}\hat{\sigma}_{1,2}, H(r_{1,2},t)]|\psi(t)\rangle. \quad (17)$$

Solving the nonlinear equations (14), (15), (17) with an accuracy to the first order in field amplitude we have

$$m_{1,2}(t)=m_{1,2}^{(0)}+\delta m_{1,2}^{(1)}(t) \quad (18)$$

where

$$m_{1,2}^{(0)}=g_{1,2}\mu_{B1,2}\langle\psi_0|\tfrac{1}{2}\hat{\sigma}_{1,2}|\psi_0\rangle \quad (19)$$

$$\delta m_1^{(1)}(t)=\frac{g_1\mu_{B1}}{\omega}[m_1^{(0)}, H_0]\sin(\omega t-k\cdot r_1)+\frac{g_1g_2\mu_{B1}\mu_{B2}}{c^2l}$$
$$\times[m_1^{(0)},[[[m_2^{(0)},H_0],n],n]]\cos[\omega t-k\cdot r_2-kl)+O(1/l^2) \quad (20)$$

$$\delta m_2^{(1)}(t)=\frac{g_1\mu_{B2}}{\omega}[m_2^{(0)}, H_0]\sin(\omega t-k\cdot r_2)+\frac{g_1g_2\mu_{B1}\mu_{B2}}{c^2l}$$
$$\times[m_2^{(0)},[[[m_1^{(0)},H_0],n],n]]\cos(\omega t-k\cdot r_1-kl)+O(1/l^2). \quad (21)$$

The substitution of (14), (15), (20), (21) into (13) and time averaging leads to:

$$\langle F_1\rangle=S(k+kn)\sin(kl+k\cdot l) \qquad \langle F_2\rangle=S(k-kn)\sin(kl-k\cdot l) \quad (22)$$

where

$$S=\frac{g_1g_2\mu_{B1}\mu_{B2}}{2c^2l}[H_0^2(m_1^{(0)},n)(m_2^{(0)},n)-(m_2^{(0)},H_0)(m_1^{(0)},n)(H_0,n)$$
$$-(m_1^{(0)},H_0)(m_2^{(0)},n)(H_0,n)+(n,H_0)^2(m_1^{(0)},m_2^{(0)})].$$

It can be seen that the structure of forces (22) is analogous to the radiative forces (7).

The presence of an external electromagnetic wave results in the appearance of long-range radiative forces between two classical or quantum magnetic moments. These time-averaged forces are square-law ones to the field amplitude and decrease inversely proportionally to the distance between the moments. This effect is analogous to the corresponding effects arising in classical acoustics.

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